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Non-uniform slot injection (suction) into steady laminar water boundary layer flow over a rotating sphere

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Abstract

The influence of non-uniform slot injection (suction) into steady axi-symmetric laminar incompressible boundary layer flows with temperature dependent viscosity and Prandtl number has been examined from the origin of the streamwise coordinate to the exact point of separation. The difficulties in obtaining the non-similar solutions at the origin of the streamwise coordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite difference scheme with the quasilinearization technique and an appropriate selection of finer step size along the streamwise coordinate. The results indicate that the separation can be delayed by non-uniform slot suction and also by moving the slot downstream but the effect of non-uniform slot injection is just the opposite. Further, the effect of variable fluid properties is to move the point of separation downstream but rotation parameter has the reverse effect.

1. Introduction

The study of flow around a rotating body has many important practical applications in problems involving fiber coating, spin-stabilized missiles, rotodynamic machine design and in the modelling of many geophysical vortices. Refs. [1–4] give informations on some of the works on flows over a rotating sphere. Most fluids have temperature-dependent properties and under circumstances where large or moderate temperature gradients exist across the fluid medium, fluid properties often vary significantly. The increase of temperature causes reduction in the viscosity across the momentum boundary layer leading to a local increase in the transport phenomena. This process has also an impact on the thermal conductivity across the thermal boundary layer which, in turn, affects the heat transfer rate at the wall. On the otherhand, the effect of variations of temperature on the density of liquids (such as water) is generally considered insignificant in the fluid flow. Recently, Saikrishnan and Roy [5] have investigated the flow and heat transfer characteristics over a rotating sphere in forced flow stream including the effect of temperature-dependent viscosity and Prandtl number.

It may be mentioned that the laminar boundary layer can sustain only a very small adverse pressure gradient without separating from a solid surface and also there are two possible modes of separation for steady three-dimensional flows, namely, singular separation and ordinary separation [6]. For singular separation, both components of the wall shear vanish simultaneously and for ordinary separation, only one component of the wall shears vanishes. Ordinary separation appears to be the dominant form of separation on most three-dimensional bodies. Excellent reviews of the phenomenon of separation of boundary layer flows have been given by Cebeci et al. [7] and Smith [8]. Fluid viscosity and thermal conductivity are the main governing fluid properties in the laminar water boundary layer forced flow and hence their variations can be expected to affect separation. As these properties are temperature dependent, variations are most easily accomplished in the boundary layer by maintaining a temperature difference between solid wall

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Nomenclature

A	dimensionless mass transfer parameter
b_1, b_2, c_1, c_2	constants
C_f, $\overline{C}_f$	skin friction coefficients in the x- and y-directions, respectively
C_p	specific heat at constant pressure
Ec	dissipation parameter
$F(=f_\eta)$	dimensionless velocity component in the x-direction
G	dimensionless temperature
k	thermal conductivity
N	(μ/μ_∞), viscosity ratio
Nu	Nusselt number
Pr	Prandtl number
$r(x)$	radius of the section normal to the axis of the sphere at a distance x
R	radius of the sphere
Re	Reynolds number
S	dimensionless velocity component in the y-direction
T	temperature
u, v, w	dimensional velocity components in x-, y- and z-directions, respectively
x, y, z	dimensional meridional, azimuthal and normal distances, respectively
$\bar{x}$	x/R, dimensionless meridional distance
$\bar{x}_0$	slot location parameter

Greek symbols

β	pressure gradient
$\Delta\eta$, $\Delta\bar{x}$	step sizes in η- and $\bar{x}$-directions, respectively
η, ξ	transformed coordinates
λ	rotation parameter
μ	dynamic viscosity
ρ	density
ψ	dimensional stream function
Ω_0	constant angular velocity
ω^*	slot length parameter

Subscripts

∞	conditions in the free stream
e, w	conditions at the edge of the boundary layer and on the surface, respectively
$\bar{x}$, η, ξ	partial derivatives with respect to these variables

and the fluid. For undersea applications, surface heating is an effective means of controlling boundary layer separation since heating promotes the stability through the interplay among thermal boundary layer, the temperature-dependent viscosity and momentum balance in the crucial region near the wall. The steady laminar boundary layer flow with temperature-dependent viscosity and Prandtl number including heat transfer for various heated bodies in water has been studied by several investigators [9–12].

It is well known that the mass transfer from a wall slot (i.e., mass injection or suction occurs in a small porous section of the body surface while there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for various potential applications including thermal protections, energizing the inner portion of boundary layer in adverse pressure gradient, and skin friction reduction on control surfaces. In fact, mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Different studies [13–15] show the effect of slot injection (suction) into laminar compressible boundary layer over a flat plate by taking the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuity at the leading and trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer distribution along a streamwise slot as has been discussed in Minkowycz et al. [16] and also in recent investigations [17,18].

The aim of the present investigation is to analyze the steady non-similar water boundary layer flow over a rotating sphere using non-uniform slot injection or suction (i.e., mass transfer occurs in a small porous section of the body surface and the remaining part of the body surface is solid). The present analysis may be useful in understanding many boundary layer flow problems of practical importance for undersea applications as would arise, for example, in suppressing recirculating bubbles and controlling transition and/or separation of the boundary layer over submerged bodies. The non-similar solutions have been obtained starting from the origin of the streamwise coordinate to the point of separation (zero skin friction in the streamwise direction) using the quasilinearization technique with an implicit finite difference scheme. There are two free parameters in this problem, one measures the length of the slot (i.e., the part of the body surface in which there is a mass transfer) and another parameter fixes the position of the slot. Thus, these two parameters help us to vary the slot length and to move the slot location.

It may be noted that the discontinuities at the leading and trailing edges of the slot have been avoided following [16–18]. Thus, the present analysis differs from those in [13–15] with finite discontinuities.

2. Analysis

Consider a steady laminar non-similar boundary layer forced convection flow (of water) with temperature-dependent viscosity and Prandtl number over a rotating sphere when the non-uniform mass transfer (suction/injection in a slot) vary with the axial distance (x) along the surface. The sphere, rotating with the constant angular velocity Ω_0, is placed in a uniform stream with its axis of rotation parallel to the free stream velocity. An orthogonal curvilinear coordinate system (see Fig. 1) has been chosen in which coordinate x measures the distance from the forward stagnation point along a meridian, y represents the distance in the direction of rotation and z is the distance normal to the body surface. The radius of a section normal to the axis of the sphere at a distance x along the meridian from the pole is $r(x)$ and it is assumed that $r(x)$ is large compared with the boundary layer thickness. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small (<40 °C). In the range of temperature considered (i.e., 0–40 °C), the variation of both density (ρ) and specific heat (C_p), of water, with temperature is less than 1% (see Table 1) and hence they are taken as constants. However, since the variations of viscosity (μ) and thermal conductivity (k) (and hence Prandtl number (Pr)) with temperature are quite significant, the viscosity and Prandtl number are assumed to vary as an inverse function of temperature (T) [5,6]:

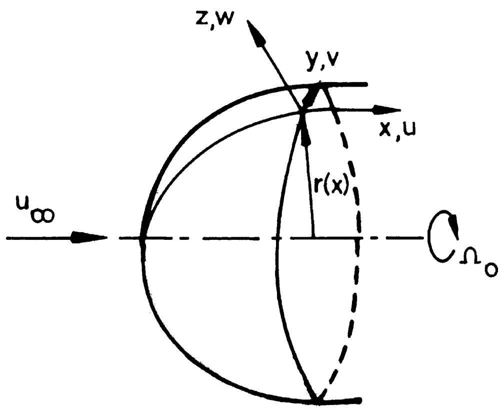

Fig. 1. Flow model and the coordinate system.

$$\mu = \frac{1}{b_1 + b_2 T} \quad \text{and} \quad Pr = \frac{1}{c_1 + c_2 T}, \tag{1}$$

where

$$b_1 = 53.41, \quad b_2 = 2.43,$$
$$c_1 = 0.068 \quad \text{and} \quad c_2 = 0.004. \tag{2}$$

The numerical data, used for these correlations, are taken from [19]. The relations (1) and (2) are reasonably good approximations for liquids such as water, particularly for small temperature differences between the wall and ambient fluid. The effect of viscous dissipation is included in the analysis. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞ and the body has a uniform temperature T_w ($T_w > T_\infty$). The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. Under the above mentioned assumptions, the boundary layer equations governing the flow can be written as [3–6]:

$$(ru)_x + (rw)_z = 0, \tag{3}$$

$$uu_x + wu_z - r^{-1}v^2 r_x = u_e(u_e)_x + \rho^{-1}(\mu u_z)_z, \tag{4}$$

$$uv_x + wv_z + uvr^{-1}r_x = \rho^{-1}(\mu v_z)_z, \tag{5}$$

$$uT_x + wT_z = \rho^{-1}\left(\frac{\mu}{Pr}T_z\right)_z + \frac{\mu}{\rho C_p}(u_z^2 + v_z^2). \tag{6}$$

The boundary conditions are given by:

$$\begin{aligned} &u(x,0) = 0, \quad v(x,0) = \Omega_0 r(x), \\ &w(x,0) = w_w(x), \quad T(x,0) = T_w = \text{constant}, \\ &u(x,\infty) = u_e(x), \quad v(x,\infty) = 0, \\ &T(x,\infty) = T_\infty = \text{constant}. \end{aligned} \tag{7}$$

Table 1
Values of thermo-physical properties of water at different temperatures [19]

Temperature (T) (°C)	Density (ρ) (g/cm^3)	Specific heat (C_p) (J × 10^7/kg K)	Thermal conductivity (k) (erg × 10^5/cm s K)	Viscosity(μ) (g × 10^{-2}/cm s)	Prandtl number, Pr
0	1.00228	4.2176	0.5610	1.7930	13.48
10	0.99970	4.1921	0.5800	1.3070	9.45
20	0.99821	4.1818	0.5984	1.0060	7.03
30	0.99565	4.1784	0.6154	0.7977	5.12
40	0.99222	4.1785	0.6305	0.6532	4.32
50	0.98803	4.1806	0.6435	0.5470	3.55

Applying the following transformations

$$\xi = \int_0^x \left(\frac{u_e}{u_\infty}\right)\left(\frac{r}{R}\right)^2 R^{-1}\,dx,$$
$$\eta = \left(\frac{Re}{2\xi}\right)^{1/2}\left(\frac{u_e}{u_\infty}\right)\left(\frac{r}{R}\right)\left(\frac{z}{R}\right),$$
$$\psi(x,z) = u_\infty R\left(\frac{2\xi}{Re}\right)^{1/2} f(\xi,\eta), \quad ur = R\frac{\partial\psi}{\partial z}, \tag{8}$$
$$wr = -R\frac{\partial\psi}{\partial x},$$
$$Re = \frac{u_\infty R\rho}{\mu_\infty}, \quad G = \frac{T-T_w}{T_\infty - T_w},$$
$$v(x,z) = \Omega_o r(x) S(\xi,\eta)$$

to Eqs. (3)–(6), we find that Eq. (3) is identically satisfied and Eqs. (4)–(6) reduce to non-dimensional form given by

$$(NF_\eta)_\eta + fF_\eta + \beta(\xi)(1-F^2) + \alpha(\xi)S^2 = 2\xi(FF_\xi - f_\xi F_\eta), \tag{9}$$

$$(NS_\eta)_\eta + fS_\eta - \alpha_1(\xi)FS = 2\xi(FS_\xi - S_\eta f_\xi), \tag{10}$$

$$(N\,Pr^{-1}G_\eta)_\eta + fG_\eta + N\mathrm{Ec}\left(\frac{u_e}{u_\infty}\right)^2[F_\eta^2 + \lambda S_\eta^2] = 2\xi(FG_\xi - f_\xi G_\eta), \tag{11}$$

where

$$N = \frac{\mu}{\mu_\infty} = \frac{b_1 + b_2T_\infty}{b_1 + b_2 T} = \frac{1}{a_1 + a_2 G},$$
$$Pr = \frac{1}{c_1 + c_2 T} = \frac{1}{a_3 + a_4 G},$$
$$a_1 = \frac{b_1 + b_2 T_w}{b_1 + b_2 T_\infty}, \quad a_2 = \frac{b_2(T_\infty - T_w)}{b_1 + b_2 T_\infty},$$
$$a_3 = c_1 + c_2 T_w, \quad a_4 = c_2(T_\infty - T_w),$$
$$\beta(\xi) = \frac{2\xi}{u_e}\frac{du_e}{d\xi}, \quad \alpha_1(\xi) = \frac{4\xi}{r}\frac{dr}{d\xi},$$
$$\lambda = \left(\frac{\Omega_o r}{u_e}\right)^2, \quad \alpha(\xi) = \frac{2\xi}{r}\frac{dr}{d\xi}\lambda,$$
$$\mathrm{Ec} = \frac{u_\infty^2}{C_p(T_\infty - T_w)},$$
$$\Delta T_w = (T_w - T_\infty), \quad u = u_e f_\eta = u_e F,$$
$$w = -\frac{ru_e}{R(2\xi R_e)^{1/2}}\left\{f + 2\xi f_\xi + \left(\beta(\xi) + \frac{\alpha_1(\xi)}{2} - 1\right)\eta F\right\}.$$

The transformed boundary conditions are

$$F(\xi,0) = 0, \quad S(\xi,0) = 1, \quad G(\xi,0) = 0,$$
$$F(\xi,\infty) = 1, \quad S(\xi,\infty) = 0, \quad G(\xi,\infty) = 1, \tag{12}$$

where $f = \int_0^\eta F\,d\eta + f_w$ and f_w is given by

$$f_w = -\xi^{-1/2}\left(\frac{Re}{2}\right)^{1/2}\int_0^{\bar{x}}\left(\frac{r}{R}\right)\frac{1}{u_\infty}w_w(\bar{x})\,d\bar{x} \tag{13}$$

The set of Eqs. (9)–(11) reduces to that of the classical non-similar flow over a stationary sphere for $\lambda = 0$. Hence Eq. (10) becomes redundant as the velocity component in the y-direction $v = 0$ (i.e., $S = 0$) for $\lambda = 0$.

In the case of a sphere of radius R, the velocity at the edge of the boundary layer and non-uniform surface mass transfer being functions of $\bar{x}$, give rise to non-similarity. The velocity at the edge of the boundary layer and the radius of revolution $r(x)$ are given by [6]

$$\frac{u_e}{u_\infty} = \frac{3}{2}\sin\bar{x}, \quad \frac{r}{R} = \sin\bar{x}, \quad \bar{x} = \frac{x}{R}.$$

Consequently, the expressions for ξ, $\beta(\xi)$, $\alpha(\xi)$, $\alpha_1(\xi)$ and f_w can be expressed as

$$\xi = \tfrac{1}{2}P_1^2P_3, \quad \beta = \tfrac{2}{3}P_3P_2^{-2}\cos\bar{x}, \quad \alpha = \lambda\beta,$$
$$\alpha_1 = 2\beta, \quad \lambda = \left(\frac{\Omega_0 r}{u_e}\right)^2 = \frac{4}{9}\left(\frac{\Omega_0 R}{u_\infty}\right)^2, \tag{14}$$

$$f_w = \begin{cases} 0, & \bar{x} \leqslant \bar{x}_0, \\ AP_1^{-1}P_3^{-1/2}C(\bar{x},\bar{x}_0), & \bar{x}_0 \leqslant \bar{x} \leqslant \bar{x}_0^*, \\ AP_1^{-1}P_3^{-1/2}C(\bar{x}_0^*,\bar{x}_0), & \bar{x} \geqslant \bar{x}_0^*, \end{cases} \tag{15}$$

where

$$C(\bar{x},\bar{x}_0) = \frac{\sin\{(\omega^*-1)\bar{x} - \omega^*\bar{x}_0\} + \sin\bar{x}_0}{(\omega^*-1)} - \frac{\sin\{(\omega^*+1)\bar{x} - \omega^*\bar{x}_0\} - \sin\bar{x}_0}{(\omega^*+1)},$$

$P_1 = 1 - \cos\bar{x}$, $P_2 = 1 + \cos\bar{x}$ and $P_3 = 2 + \cos\bar{x}$.

Here, $w_w(\bar{x})$ (in (13)) is taken as

$$w_w(\bar{x}) = \begin{cases} -u_\infty\left(\frac{Re}{2}\right)^{-1/2}2^{1/2}A\sin\{\omega^*(\bar{x}-\bar{x}_0)\}, \\ \bar{x}_0 \leqslant \bar{x} \leqslant \bar{x}_0^*, 0, \quad \bar{x} \leqslant \bar{x}_0 \quad \text{and} \quad \bar{x} \geqslant \bar{x}_0^*, \end{cases}$$

where ω^* and $\bar{x}_0$ are the two free parameters which determine the slot length and slot location. The function $w_w(\bar{x})$ is continuous for all values of $\bar{x}$ and it has a non-zero value only in the interval $[\bar{x}_0, \bar{x}_0^*]$. The reason for taking such a function is that it allows the mass transfer to change slowly in the neighbourhood of leading and trailing edges of the slot. The parameter $A > 0$ or $A < 0$ according to whether there is a suction or an injection. It is convenient to express Eqs. (9)–(11) in terms of $\bar{x}$ instead of ξ. Eq. (14) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi} = B(\bar{x})\frac{\partial}{\partial\bar{x}}, \tag{16}$$

where $B(\bar{x}) = 3^{-1}\tan(\bar{x}/2)P_3P_2^{-1}$.

Substituting Eq. (16) in Eqs. (9)–(11), we obtain

$$(NF_\eta)_\eta + fF_\eta + \beta(\bar{x})(1 - F^2) + \alpha(\bar{x})S^2 = 2B(\bar{x})(FF_{\bar{x}} - f_{\bar{x}}F_\eta), \quad (17)$$

$$(NS_\eta)_\eta + fS_\eta - \alpha_1(\bar{x})FS = 2B(\bar{x})(FS_{\bar{x}} - S_\eta f_{\bar{x}}), \quad (18)$$

$$(N\,Pr^{-1}G_\eta)_\eta + fG_\eta + N\mathrm{Ec}\left(\frac{u_\mathrm{e}}{u_\infty}\right)^2[F_\eta^2 + \lambda S_\eta^2] = 2B(\bar{x})(FG_{\bar{x}} - f_{\bar{x}}G_\eta), \quad (19)$$

where $N = 1/(a_1 + a_2G)$, and $Pr = 1/(a_3 + a_4G)$.

The boundary conditions become

$$F(\bar{x},0) = 0, \quad S(\bar{x},0) = 1, \quad G(\bar{x},0) = 0, \qquad F(\bar{x},\infty) = 1, \quad S(\bar{x},\infty) = 0, \quad G(\bar{x},\infty) = 1, \quad (20)$$

where $f = \int_0^\eta F\,\mathrm{d}\eta + f_\mathrm{w}$.

The skin friction coefficients in x- and y-directions can be expressed in the form:

$$C_\mathrm{f}(Re)^{1/2} = \tfrac{9}{2}\sin\bar{x}\,P_2P_3^{-1/2}N_\mathrm{w}(F_\eta)_\mathrm{w}, \quad (21)$$

$$\overline{C}_\mathrm{f}(Re)^{1/2} = \tfrac{9}{2}\lambda^{1/2}\sin\bar{x}\,P_2P_3^{-1/2}N_\mathrm{w}(S_\eta)_\mathrm{w}. \quad (22)$$

Similarly, the heat transfer coefficient in terms of Nusselt number can be written as

$$Nu(Re)^{-1/2} = \tfrac{3}{2}P_2P_3^{-1/2}(G_\eta)_\mathrm{w}, \quad (23)$$

where

$$C_\mathrm{f} = 2\frac{[\mu(\frac{\partial u}{\partial z})]_\mathrm{w}}{\rho u_\infty^2}, \quad \overline{C}_\mathrm{f} = 2\frac{[\mu(\frac{\partial v}{\partial z})]_\mathrm{w}}{\rho u_\infty^2}, \quad Nu = \frac{R(\frac{\partial T}{\partial z})_\mathrm{w}}{(T_\infty - T_\mathrm{w})},$$

and

$$N_\mathrm{w} = \frac{1}{a_1 + a_2G_\mathrm{w}} = \text{constant}.$$

3. Method of solution

Applying the quasilinearization technique [20,21], we replace the non-linear partial differential equations (17)–(19) by an iterative sequence of linear equations as follows:

$$X_1^kF_{\eta\eta}^{k+1} + X_2^kF_\eta^{k+1} + X_3^kF^{k+1} + X_4^kF_{\bar{x}}^{k+1} + X_5^kG_\eta^{k+1} + X_6^kG^{k+1} + X_7^kS^{k+1} = X_8^k, \quad (24)$$

$$Y_1^kS_{\eta\eta}^{k+1} + Y_2^kS_\eta^{k+1} + Y_3^kS^{k+1} + Y_4^kS_{\bar{x}}^{k+1} + Y_5^kF^{k+1} + Y_6^kG_\eta^{k+1} + Y_7^kG^{k+1} = Y_8^k, \quad (25)$$

$$Z_1^kG_{\eta\eta}^{k+1} + Z_2^kG_\eta^{k+1} + Z_3^kG^{k+1} + Z_4^kG_{\bar{x}}^{k+1} + Z_5^kF_\eta^{k+1} + Z_6^kF + Z_7^kS_\eta^{k+1} = Z_8^k, \quad (26)$$

where the coefficient functions with iterative index k are known and functions with iterative index $k+1$ are to be determined.

The boundary conditions become

$$F^{k+1} = 0, \quad S^{k+1} = 1, \quad G^{k+1} = 0 \quad \text{at } \eta = 0,$$
$$F^{k+1} = 1, \quad S^{k+1} = 0, \quad G^{k+1} = 1 \quad \text{at } \eta = \eta_\infty,$$

where η_∞ is the edge of the boundary layer. The coefficients in Eqs. (24)–(26) are given by

$$X_1^k = N,$$
$$X_2^k = -a_1N^2G_\eta + f + 2B(\bar{x})f_{\bar{x}},$$
$$X_3^k = -2\beta F - 2B(\bar{x})F_{\bar{x}},$$
$$X_4^k = -2B(\bar{x})F,$$
$$X_5^k = -a_1N^2F_\eta,$$
$$X_6^k = -a_1N^2F_{\eta\eta} + 2a_1^2N^3F_\eta G_\eta,$$
$$X_7^k = 2\alpha S,$$
$$X_8^k = (1 - N^{-1})N^2F_{\eta\eta} + a_1N^2F_\eta G_\eta - 2a_1N^3F_\eta G_\eta - \beta(1 + F^2) + \alpha S^2 - 2B(\bar{x})FF_{\bar{x}},$$
$$Y_1^k = N,$$
$$Y_2^k = -a_1N^2G_\eta + f + 2B(\bar{x})f_{\bar{x}},$$
$$Y_3^k = -\alpha_1F,$$
$$Y_4^k = -2B(\bar{x})F,$$
$$Y_5^k = -\alpha_1S - 2B(\bar{x})S_{\bar{x}},$$
$$Y_6^k = -a_1N^2S_\eta,$$
$$Y_7^k = -a_1N^2S_{\eta\eta} + 2a_1^2N^3S_\eta G_\eta,$$
$$Y_8^k = (1 - N^{-1})N^2S_{\eta\eta} + a_1N^2S_\eta G_\eta - 2a_1N^3S_\eta G_\eta - \alpha_1FS - 2B(\bar{x})FS_{\bar{x}},$$
$$Z_1^k = N\,Pr^{-1},$$
$$Z_2^k = 2a_3NG_\eta - 2a_1N^2\,Pr^{-1}G_\eta + f + 2B(\bar{x})f_{\bar{x}},$$
$$Z_3^k = -a_1N^2\,Pr^{-1}G_{\eta\eta} + a_3NG_{\eta\eta} + 2a_1^2N^3\,Pr^{-1}G_\eta^2 - 2a_1a_3N^2G_\eta^2 - a_1\mathrm{Ec}\left(\frac{u_\mathrm{e}}{u_\infty}\right)^2N^2F_\eta^2 - a_1\mathrm{Ec}\lambda\sin^2\bar{x}N^2S_\eta^2,$$
$$Z_4^k = -2B(\bar{x})F,$$
$$Z_5^k = 2\mathrm{Ec}\left(\frac{u_\mathrm{e}}{u_\infty}\right)^2NF_\eta,$$
$$Z_6^k = -2B(\bar{x})G_{\bar{x}},$$
$$Z_7^k = 2\mathrm{Ec}\lambda\sin^2\bar{x}NS_\eta,$$
$$Z_8^k = a_3NG_{\eta\eta}G + (1 - N^{-1})N^2\,Pr^{-1}G_{\eta\eta} + \mathrm{Ec}\left(\frac{u_\mathrm{e}}{u_\infty}\right)^2NF_\eta^2 + \mathrm{Ec}\lambda\sin^2\bar{x}NS_\eta^2 + \mathrm{Ec}\left(\frac{u_\mathrm{e}}{u_\infty}\right)^2(1 - N^{-1})N^2F_\eta^2 + \mathrm{Ec}\lambda\sin^2\bar{x}(1 - N^{-1})N^2S_\eta^2 + a_1N^2\,Pr^{-1}G_\eta^2 - 2a_1N^3\,Pr^{-1}G_\eta^2 + 2a_3N^2G_\eta^2 - a_3NG_\eta^2 - 2B(\bar{x})FG_{\bar{x}}.$$

Now, the resulting sequence of linear partial differential equations (24)–(26) were expressed in difference form using central difference scheme in η-direction and backward difference scheme in $\bar{x}$-direction. In each iteration step, the equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is solved using Varga's algorithm [22]. The step size in the η-direction has been chosen as $\Delta\eta = 0.01$ throughout the computations as it has been found that a further decrease in $\Delta\eta$ does not change the results up to the fourth decimal place. In the $\bar{x}$-direction, $\Delta\bar{x} = 0.01$ has been used for small values of $\bar{x}(\leqslant 0.5)$, then it has been decreased to $\Delta\bar{x} = 0.005$. This value of $\Delta\bar{x}$ has been used for $\bar{x} \leqslant 1.20$, thereafter the step size has been reduced further, ultimately choosing a value $\Delta\bar{x} = 0.0001$ in the neighbourhood of the point of zero skin friction. This has been done because the convergence becomes slower when the point of vanishing meridional skin friction is approached. A convergence criterion based on the relative difference between the current and the previous iterations has been used. The solution is assumed to have converged and the iterative process is terminated when

$$\mathrm{Max}\left\{|(F_\eta)_w^{k+1} - (F_\eta)_w^k|,\ |(S_\eta)_w^{k+1} - (S_\eta)_w^k|,\ |(G_\eta)_w^{k+1} - (G_\eta)_w^k|\right\} < 10^{-4}.$$

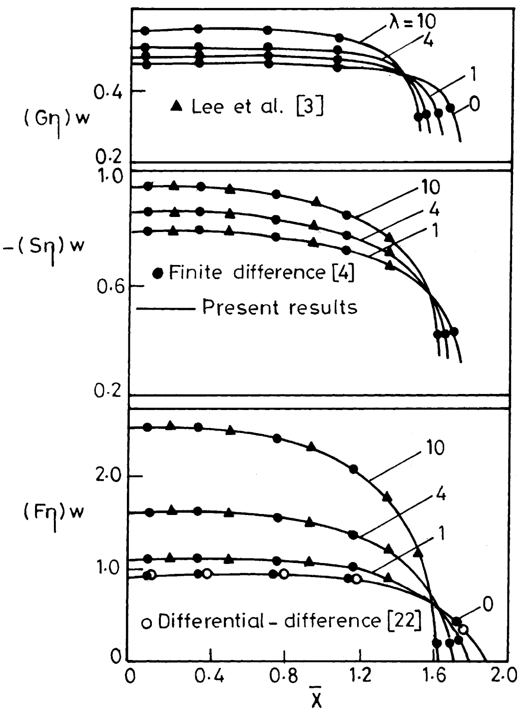

Fig. 2. Comparison of skin friction and heat transfer parameters.

4. Results and discussion

Computations have been carried out for various values of $\lambda(0 \leqslant \lambda \leqslant 10)$, $\mathrm{Ec}(-0.4 \leqslant \mathrm{Ec} \leqslant 0)$, $A(-1.0 \leqslant A \leqslant 1.25)$, $\bar{x}_0(0.5 \leqslant \bar{x}_0 \leqslant 1.25)$ and $\Delta T_w(0 \leqslant \Delta T_w \leqslant 15)$. In order to assess the accuracy of the procedure, solutions have been obtained for the incompressible flow cases and comparisons are displayed in Fig. 2. The skin friction and heat transfer parameters $((F_\eta)_w, (G_\eta)_w)$ for the stationary sphere ($\lambda = 0$) have been compared with those of the differential-difference method [23] and finite difference method [4] (see Fig. 2). The results are found to be in good agreement. The skin friction and the heat transfer parameters $((F_\eta)_w, -(S_\eta)_w, (G_\eta)_w)$ corresponding to a rotating sphere ($\lambda > 0$) for $\mathrm{Ec} = 0$ have also been compared with those of Lee et al. [3], Kumari and Nath [4] (see Fig. 2). The results are found to be in excellent agreement with the present results. With a view to compare the results corresponding to variable and constant fluid properties without mass transfer, the governing non-dimensional equations (17)–(19) with boundary conditions (20) have also been solved for $N = 1$, $Pr = 7.0$, $\mathrm{Ec} = -0.25$ with different values of $\lambda(0 \leqslant \lambda \leqslant 10)$. The skin friction coefficients in the x- and y-directions $(C_f(Re)^{1/2}, \overline{C}_f(Re)^{1/2})$ and heat transfer coefficient $(Nu(Re)^{-1/2})$ have been compared with the recent result of Saikrishnan and Roy [5] which shows a good agreement with the present results but the comparisons are not shown here to limit the number of figures.

The effects of non-uniform slot suction (or injection) parameter ($A > 0$ or $A < 0$) and $\bar{x}_0$ (which fixes the slot location) on the skin friction and heat transfer coefficients $[C_f(Re)^{1/2}, \overline{C}_f(Re)^{1/2}, Nu(Re)^{-1/2}]$ are presented in Figs. 3 and 4. In the case of non-uniform slot suction, the skin friction and heat transfer coefficients $[C_f(Re)^{1/2}, \overline{C}_f(Re)^{1/2}, Nu(Re)^{-1/2}]$ increase as slot starts and attain their maximum values before the trailing edge of the slot. Finally, $C_f(Re)^{1/2}$, $\overline{C}_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ decrease from their maximum values and $C_f(Re)^{1/2}$ reaches zero but $\overline{C}_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ remain finite (Fig. 3). This implies (as mentioned earlier) that ordinary separation occurs at this point. For the problem under consideration, singular separation does not occur (i.e., for no value of $\bar{x}$, $C_f(Re)^{1/2}$ and $\overline{C}_f(Re)^{1/2}$ reach zero value simultaneously). Hence, in subsequent discussions, for the sake of convenience, we have used the word separation to denote ordinary separation. The results indicate that the effect of non-uniform slot suction is to move the point of separation downstream, i.e., it delays the separation, but the injection through a slot on the body surface has the reverse effect as shown in Fig. 4. It is also observed (see Fig. 3) that if we move the location of the slot downstream, the point of separation also moves downstream (i.e., it delays the separation). Thus, separation can be delayed by non-uniform slot suction

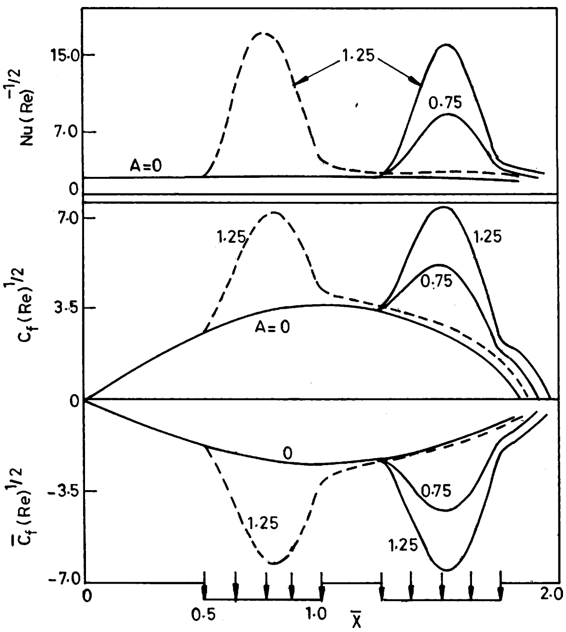

Fig. 3. Effects of suction ($A > 0$) and slot location ($\bar{x}_0$) on skin friction and heat transfer coefficients for $T_\infty = 18.7$ °C, $\Delta T_w = 10.0$ °C, Ec = 0.0, $\lambda = 1$ and $\omega^* = 2\pi$. (—) slot location at $\bar{x}_0 = 1.25$, (– – –) slot location at $\bar{x}_0 = 0.5$.

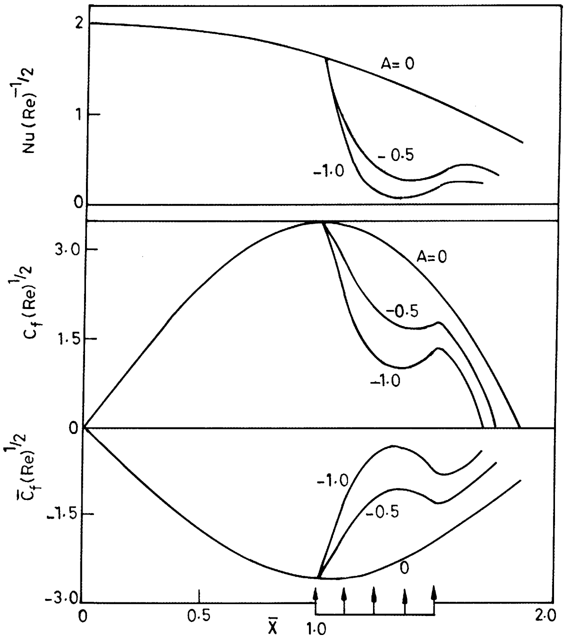

Fig. 4. Effect of injection ($A < 0$) on the skin friction and heat transfer coefficients for $T_\infty = 18.7$ °C, $\Delta T_w = 10.0$ °C, Ec = 0, $\bar{x}_0 = 1.0$, $\lambda = 1$ and $\omega^* = 2\pi$.

($A > 0$) and also by moving the slot downstream. To be more specific, for Ec = 0, $\lambda = 1$, $\omega^* = 2\pi$ and $\bar{x}_0 = 1.25$ (Fig. 3), the point of separation moves downstream approximately by 7% as the rate of suction ($A > 0$) increases from 0 to 1.25.

Fig. 5 displays the effect of viscous dissipation and rotation parameters (Ec, λ) on temperature profile (G), and heat transfer coefficient ($Nu(Re)^{-1/2}$). It is observed from Fig. 5 that for Ec $\neq 0$, the temperature profile G first decrease to negative values and then tends to 1 asymptotically. This implies that the temperature of the fluid near the wall is greater than that at the wall. Due to viscous dissipation, the fluid near the wall heats up and its temperature becomes more than the wall, although the wall is maintained at constant higher temperature [$T_\infty = 18.7$ °C, $T_w = 28.7$ °C]. Thus, the cooler free stream is unable to cool the hot wall due to the "heat cushion" provided by the frictional heating. Therefore, the wall instead of being cooled gets heated. This effect becomes more pronounced as the rotation parameter λ increases. Further, it is noticed that skin friction and heat transfer coefficients [$C_f(Re)^{1/2}$, $\bar{C}_f(Re)^{1/2}$, $Nu(Re)^{-1/2}$] decrease with the increase of the dissipation parameter Ec and the effect of viscous dissipation is

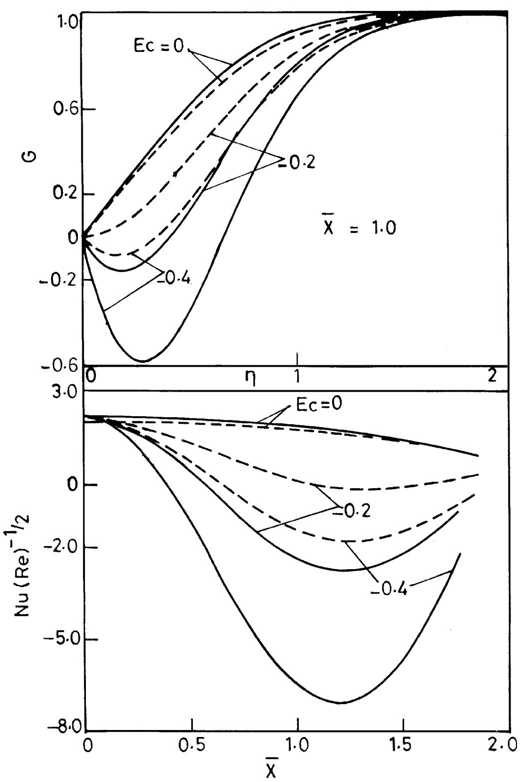

Fig. 5. Effect of viscous dissipation and rotation parameters (Ec, λ) on temperature profile G and heat transfer coefficient $Nu(Re)^{-1/2}$ for $A = 0$, $\Delta T_w = 10.0$ °C and $T_\infty = 18.7$ °C. (—) $\lambda = 4$, (– – –) $\lambda = 1$.

more significant on the heat transfer rate than on skin friction. In particular for $\lambda = 1$, it is found that the percentage decrease of $Nu(Re)^{-1/2}$ for the change of values in Ec from 0 to -0.4 at $\bar{x} = 1.0$ is 198% whereas the percentage decrease in the values of skin friction coefficients $[C_f(Re)^{1/2}, \overline{C}_f(Re)^{1/2}]$ are within 2% for the change of Ec from 0 to -0.4 at $\bar{x} = 1.0$ and only heat transfer coefficient $(Nu(Re)^{-1/2})$ is presented in Fig. 5 to reduce the number of figures. Further, it is noticed in Fig. 5 that for $\mathrm{Ec} \neq 0$, $Nu(Re)^{-1/2}$ becomes negative indicating reversal of the direction of heat transfer from the initial wall to fluid, to fluid to wall (which can also be understood by the corresponding undershoot in temperature profiles). Similar trend has been observed in the case of constant fluid properties [4] as well as variable fluid properties [5,12]. However, in the absence of viscous dissipation ($\mathrm{Ec} = 0$) heat transfer takes place in the usual way (from wall to the fluid).

5. Conclusions

Non-similar solution of a steady laminar incompressible (water) boundary layer flow over a rotating sphere with non-uniform slot injection (suction) has been obtained starting from the origin of streamwise coordinate to the exact point of separation (ordinary separation) The results pertaining to the present study indicate that the separation can be delayed by non-uniform slot suction and also by moving the slot downstream but the non-uniform slot injection has the opposite effect. Also, the effect of variable fluid properties is to move the point of separation downstream but the rotation parameter has the reverse effect. The heat transfer rate is found to depend strongly on viscous dissipation, but it has comparatively very less effect on the skin frictions and hence on the point of separation.

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